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MARKSCHEME

November 2010

MATHEMATICS

Standard Level

Paper 1

18 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL : Guidance for e-marking November 2010**". It is **essential** that you read this document before you start marking. In particular, please note the following.

Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the 'must be seen' marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.

All the marks will be added and recorded by scoris. Do **not** enter marks directly into the mark entry box.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note. (Example 1)
- Once a correct answer to a question or part-question is seen, ignore further working.

3 ***N*** **marks**

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown **(M, A, R)**.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 **Implied and must be seen marks**

*Implied marks appear in **brackets e.g. (M1)**.*

- Implied marks can only be awarded if **correct** work is seen or if implied in subsequent working (a correct answer does not necessarily mean that the implied marks are all awarded).
- Normally the correct work is seen or implied in the next line.
- Where there is an ***(M1)*** followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the ***(M1)***. (Example 2)

*Must be seen marks appear without **brackets e.g. M1**.*

- Must be seen marks can only be awarded if **appropriate** work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 **Follow through marks (only applied after an error is made)**

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate.)
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value. (Example 3)
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts. (Example 3)

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question. Award the marks as usual and then stamp **MR** against the answer. Scoris will automatically deduct 1 mark from the question total. A candidate should be penalized only once for a particular mis-read. Do not stamp **MR** again for that question, unless the candidate makes another mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled **(d)** and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen). (Example 4)

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Candidates should be penalized ***once only IN THE PAPER*** *for an accuracy error* ***(AP)****. Award the marks as usual then stamp* ***(AP)*** *against the answer. Scoris will automatically deduct 1 mark from the paper total.* Please see section E in the guidance document which clearly explains, with the use of screenshots, how this works in scoris.

- If a final correct answer is incorrectly rounded, apply the ***AP***.
- If the level of accuracy is not specified in the question, apply the ***AP*** for correct final answers not given to three significant figures.
- Intermediate values are sometimes written as 3.24(741). This indicates that using 3.24 (or 3.25) is acceptable, but the more accurate value is 3.24741. The digits in brackets are not required for the marks. If candidates work with fewer than three significant figures, this could lead to an ***AP***.
- Do not accept unfinished numerical answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*e.g.* 6/8).

If there is no working shown, and answers are given to the correct two significant figures, apply the ***AP*** with the ***N*** marks for correct two significant figures answers. However, do **not** accept answers to one significant figure without working.

11 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

12 Style

The markscheme aims to present answers using good communication, e.g. if the question asks to find the value of k, the markscheme will say $k = 3$*, but the marks will be for the correct value 3 – there is usually no need for the "* $k =$ *". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, e.g. if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".*

The markscheme often uses words to describe what the marks are for, followed by examples, using the e.g. notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable.* (Example 5)

13 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

EXAMPLES

Please check the references in the instructions above.

EXAMPLE 1

(a) evidence of using $\sum p_i = 1$ ***(M1)***

correct substitution ***A1***

e.g. $10k^2 + 3k + 0.6 = 1,\ 10k^2 + 3k - 0.4 = 0$

$k = 0.1$ ***A2*** ***N2***

Note: Award ***A1*** for a final answer of $k = 0.1,\ k = -0.4$.

(b) evidence of using $\mathrm{E}(X) = \sum p_i x_i$ ***(M1)***

correct substitution ***(A1)***

e.g. $-1 \times 0.2 + 2 \times 0.4 + 3 \times 0.3$

$\mathrm{E}(X) = 1.5$ ***A1*** ***N2***

Note: Award ***FT*** marks only on values of *k* between 0 and 1.

EXAMPLE 2

(a) intercepts when $f(x) = 0$ ***(M1)***

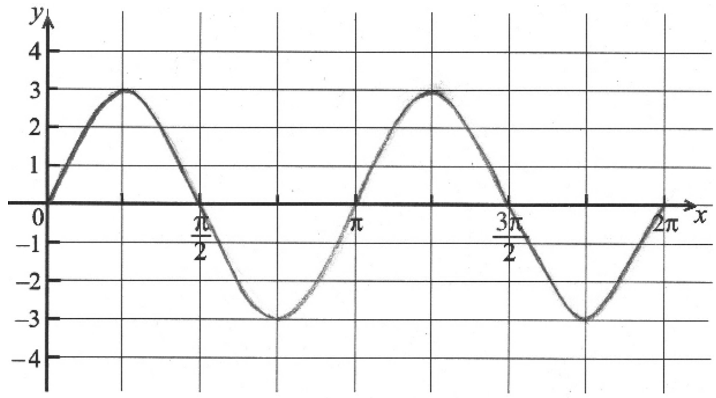
Note: 1 correct answer seen is sufficient evidence to award the ***(M1)***.

(1.54, 0) (4.13, 0) (accept $x = 1.54 \quad x = 4.13$) ***A1A1*** ***N3***

EXAMPLE 3

(a) period $= \pi$ (accept 180°) *A1* *N1*

(b)



A1A1A1 *N3*

Notes: Award ***A1*** for amplitude 3, ***A1*** for **their** period, ***A1*** for a sine curve through $(0, 0)$ and $(0, 2\pi)$.
If the answer to part (a) is incorrect, but the graph period is correct, award ***A1***.

(c) evidence of appropriate approach *(M1)*
e.g. line $y = 2$ on graph, discussion of number of solutions in the domain
4 (solutions) *A1* *N2*

Notes: Award the ***N2*** for 4 solutions, even if this is inconsistent with their graph.
Award ***M1A1 FT*** for incorrect answers which are consistent with their graph.

EXAMPLE 4

for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(= 10\cos(5x-3)\right)$ *A1*

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

EXAMPLE 5

(i) evidence of approach *M1*
e.g. $\vec{AO} + \vec{OB} = \vec{AB}$, $B - A$

SECTION A

QUESTION 1

(a) $r = \frac{16}{32}\left(= \frac{1}{2}\right)$ *A1* *N1*

(b) correct calculation or listing terms *(A1)*

e.g. $32 \times \left(\frac{1}{2}\right)^{6-1}$, $8 \times \left(\frac{1}{2}\right)^{3}$, $32, \ldots 4, 2, 1$

$u_6 = 1$ *A1* *N2*

(c) evidence of correct substitution in S_∞ *A1*

e.g. $\frac{32}{1-\frac{1}{2}}$, $\frac{32}{\frac{1}{2}}$

$S_\infty = 64$ *A1* *N1*

[5 marks]

QUESTION 2

(a) evidence of choosing the product rule *(M1)*

e.g. $uv' + vu'$

correct derivatives $\cos x$, 2 *(A1)(A1)*

$g'(x) = 2x\cos x + 2\sin x$ *A1* *N4*

(b) attempt to substitute into gradient function *(M1)*

e.g. $g'(\pi)$

correct substitution *(A1)*

e.g. $2\pi\cos\pi + 2\sin\pi$

gradient $= -2\pi$ *A1* *N2*

[7 marks]

QUESTION 3

(a) correct substitution in $l = r\theta$ ***(A1)***

e.g. $10 \times \frac{\pi}{3}$, $\frac{1}{6} \times 2\pi \times 10$

arc length $= \frac{20\pi}{6}$ $\left(= \frac{10\pi}{3}\right)$ ***A1*** ***N2***

(b) area of large sector $= \frac{1}{2} \times 10^2 \times \frac{\pi}{3}$ $\left(= \frac{100\pi}{6}\right)$ ***(A1)***

area of small sector $= \frac{1}{2} \times 8^2 \times \frac{\pi}{3}$ $\left(= \frac{64\pi}{6}\right)$ ***(A1)***

evidence of valid approach (seen anywhere) ***M1***

e.g. subtracting areas of two sectors, $\frac{1}{2} \times \frac{\pi}{3}(10^2 - 8^2)$

area shaded $= 6\pi$ $\left(\text{accept } \frac{36\pi}{6}, \textit{etc.}\right)$ ***A1*** ***N3***

[6 marks]

QUESTION 4

(a) $p = \frac{4}{5}$ ***A1*** ***N1***

(b) multiplying along the branches ***(M1)***

e.g. $\frac{1}{5} \times \frac{1}{4}$, $\frac{12}{40}$

adding products of probabilities of two mutually exclusive paths ***(M1)***

e.g. $\frac{1}{5} \times \frac{1}{4} + \frac{4}{5} \times \frac{3}{8}$, $\frac{1}{20} + \frac{12}{40}$

$\mathrm{P}(B) = \frac{14}{40}$ $\left(= \frac{7}{20}\right)$ ***A1*** ***N2***

(c) appropriate approach which must include A' (may be seen on diagram) ***(M1)***

e.g. $\frac{\mathrm{P}(A' \cap B)}{\mathrm{P}(B)}$ $\left(\text{do not accept } \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}\right)$

$$\mathrm{P}(A' \mid B) = \frac{\frac{4}{5} \times \frac{3}{8}}{\frac{7}{20}}$$ ***(A1)***

$\mathrm{P}(A' \mid B) = \frac{12}{14}$ $\left(= \frac{6}{7}\right)$ ***A1*** ***N2***

[7 marks]

QUESTION 5

(a) attempt to substitute $1-2\sin^2\theta$ for $\cos 2\theta$ *(M1)*
correct substitution *A1*
e.g. $4-(1-2\sin^2\theta)+5\sin\theta$
$4-\cos 2\theta+5\sin\theta = 2\sin^2\theta+5\sin\theta+3$ *AG* *N0*

(b) evidence of appropriate approach to solve *(M1)*
e.g. factorizing, quadratic formula

correct working *A1*
e.g. $(2\sin\theta+3)(\sin\theta+1)$, $(2x+3)(x+1)=0$, $\sin x = \frac{-5\pm\sqrt{1}}{4}$

correct solution $\sin\theta=-1$ $\left(\text{do not penalise for including } \sin\theta=-\frac{3}{2}\right)$ *(A1)*

$\theta = \frac{3\pi}{2}$ *A2* *N3*

[7 marks]

QUESTION 6

evidence of integration
e.g. $f(x)=\int \sin(2x-3)\,\mathrm{d}x$ *(M1)*
$= -\frac{1}{2}\cos(2x-3)+C$ *A1A1*

substituting initial condition into **their** expression (even if C is missing) *M1*
e.g. $4 = -\frac{1}{2}\cos 0 + C$
$C=4.5$ *(A1)*

$f(x) = -\frac{1}{2}\cos(2x-3)+4.5$ *A1* *N5*

[6 marks]

QUESTION 7

(a) correct substitution into the formula for the determinant *(A1)*

e.g. $\det \boldsymbol{A} = 9e^{x} \times e^{3x} - e^{x} \times e^{x}$

$\det \boldsymbol{A} = 9e^{4x} - e^{2x}$ ***A1*** ***N2***

(b) recognizing that no inverse implies $\det \boldsymbol{A} = 0$ ***R1***

e.g. $9e^{4x} - e^{2x} = 0$, $ad - bc = 0$

attempt to solve equation ***(M1)***

e.g. $e^{2x} = \frac{1}{9}$, $e^{-2x} = 9$, $e^{2x}(9e^{2x} - 1) = 0$, $9e^{4x} = e^{2x}$

rearranging to get correct log equation ***(A1)***

e.g. $2x = \ln\frac{1}{9}$, $-2x = \ln 9$, $\ln(9e^{4x}) = \ln(e^{2x})$

isolating x ***A1***

e.g. $x = \frac{1}{2}\ln\frac{1}{9}$, $x = -\frac{1}{2}\ln 9$, $x = \ln\frac{1}{3}$, $a = -\frac{1}{2}$, $b = 9$

$x = -\ln 3$ (accept $a = -1$, $b = 3$) ***A1*** ***N3***

[7 marks]

SECTION B

QUESTION 8

(a) (i) correct approach *A1*

e.g. $\vec{OC} - \vec{OA}$, $\begin{pmatrix} 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\vec{AC} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ *AG* *N0*

(ii) appropriate approach *(M1)*

e.g. $D - B$, $\begin{pmatrix} 4 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \end{pmatrix}$, move 3 to the right and 6 down

$\vec{BD} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$ *A1* *N2*

(iii) finding the scalar product *A1*

e.g. $4(3) + 2(-6)$, $12 - 12$

valid reasoning *R1*

e.g. $4(3) + 2(-6) = 0$, scalar product is zero

$\vec{AC}$ is perpendicular to $\vec{BD}$ *AG* *N0*

[5 marks]

(b) (i) correct "position" vector for $\boldsymbol{u}$; "direction" vector for $\boldsymbol{v}$ *A1A1* *N2*

e.g. $\boldsymbol{u} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$, $\boldsymbol{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $\boldsymbol{v} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$, $\boldsymbol{v} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

accept in equation *e.g.* $\begin{pmatrix} 5 \\ 2 \end{pmatrix} + t\begin{pmatrix} -4 \\ -2 \end{pmatrix}$

(ii) any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$, where $\boldsymbol{b} = \vec{BD}$

e.g. $\boldsymbol{r} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} + t\begin{pmatrix} 3 \\ -6 \end{pmatrix}$, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + t\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ *A2* *N2*

[4 marks]

continued …

Question 8 continued

(c) **METHOD 1**

substitute $(3, k)$ into equation for (AC) or (BD) ***(M1)***

e.g. $3 = 1 + 4s$, $3 = 1 + 3t$

value of t or s ***A1***

e.g. $s = \frac{1}{2}, -\frac{1}{2}$, $t = \frac{2}{3}, -\frac{1}{3}$

substituting ***A1***

e.g. $k = 0 + \frac{1}{2}(2)$,

$k = 1$ ***AG*** ***N0***

[3 marks]

METHOD 2

setting up two equations ***(M1)***

e.g. $1 + 4s = 4 + 3t$, $2s = -1 - 6t$; setting vector equations of lines equal

value of t or s ***A1***

e.g. $s = \frac{1}{2}, -\frac{1}{2}$, $t = \frac{2}{3}, -\frac{1}{3}$

substituting ***A1***

e.g. $\boldsymbol{r} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 3 \\ -6 \end{pmatrix}$,

$k = 1$ ***AG*** ***N0***

[3 marks]

(d) $\overrightarrow{\text{PD}} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ ***(A1)***

$|\overrightarrow{\text{PD}}| = \sqrt{2^2 + 1^2}$ $\left(= \sqrt{5}\right)$ ***(A1)***

$|\overrightarrow{\text{AC}}| = \sqrt{4^2 + 2^2}$ $\left(= \sqrt{20}\right)$ ***(A1)***

area $= \frac{1}{2} \times |\overrightarrow{\text{AC}}| \times |\overrightarrow{\text{PD}}|$ $\left(= \frac{1}{2} \times \sqrt{20} \times \sqrt{5}\right)$ ***M1***

$= 5$ ***A1*** ***N4***

[5 marks]

Total [17 marks]

QUESTION 9

(a) attempt to form composition (in any order) *(M1)*

$(f \circ g)(x) = (x-1)^2 + 4 \quad (x^2 - 2x + 5)$ *A1* *N2*

[2 marks]

(b) **METHOD 1**

vertex of $f \circ g$ at $(1, 4)$ *(A1)*

evidence of appropriate approach *(M1)*

e.g. adding $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ to the coordinates of the vertex of $f \circ g$

vertex of h at $(4, 3)$ *A1* *N3*

[3 marks]

METHOD 2

attempt to find $h(x)$ *(M1)*

e.g. $\left((x-3)-1\right)^2 + 4 - 1$, $h(x) = (f \circ g)(x-3) - 1$

$h(x) = (x-4)^2 + 3$ *(A1)*

vertex of h at $(4, 3)$ *A1* *N3*

[3 marks]

(c) evidence of appropriate approach *(M1)*

e.g. $(x-4)^2 + 3$, $(x-3)^2 - 2(x-3) + 5 - 1$

simplifying *A1*

e.g. $h(x) = x^2 - 8x + 16 + 3$, $x^2 - 6x + 9 - 2x + 6 + 4$

$h(x) = x^2 - 8x + 19$ *AG* *N0*

[2 marks]

continued …

Question 9 continued

(d) **METHOD 1**

equating functions to find intersection point *(M1)*
e.g. $x^2 - 8x + 19 = 2x - 6$, $y = h(x)$

$x^2 - 10x + 25 = 0$ ***A1***

evidence of appropriate approach to solve *(M1)*
e.g. factorizing, quadratic formula

appropriate working ***A1***
e.g. $(x-5)^2 = 0$

$x = 5$ $(p = 5)$ ***A1*** ***N3***
[5 marks]

METHOD 2

attempt to find $h'(x)$ *(M1)*
$h'(x) = 2x - 8$ ***A1***

recognizing that the gradient of the tangent is the derivative *(M1)*
e.g. gradient at $p = 2$

$2x - 8 = 2$ $(2x = 10)$ ***A1***

$x = 5$ ***A1*** ***N3***
[5 marks]

Total [12 marks]

QUESTION 10

(a) (i) substitute into gradient $= \frac{y_1 - y_2}{x_1 - x_2}$ ***(M1)***

e.g. $\frac{f(a) - 0}{a - \frac{2}{3}}$

substituting $f(a) = a^3$

e.g. $\frac{a^3 - 0}{a - \frac{2}{3}}$ ***A1***

gradient $= \frac{a^3}{a - \frac{2}{3}}$ ***AG*** ***N0***

(ii) correct answer ***A1*** ***N1***

e.g. $3a^2$, $f'(a) = 3$, $f'(a) = \frac{a^3}{a - \frac{2}{3}}$

(iii) **METHOD 1**

evidence of approach ***(M1)***

e.g. $f'(a) =$ gradient, $3a^2 = \frac{a^3}{a - \frac{2}{3}}$

simplify ***A1***

e.g. $3a^2\left(a - \frac{2}{3}\right) = a^3$

rearrange ***A1***

e.g. $3a^3 - 2a^2 = a^3$

evidence of solving ***A1***

e.g. $2a^3 - 2a^2 = 2a^2(a-1) = 0$

$a = 1$ ***AG*** ***N0***

continued ...

Question 10 continued

METHOD 2

gradient RQ $= \dfrac{-8}{-2-\frac{2}{3}}$ *A1*

simplify *A1*

e.g. $\dfrac{-8}{-\frac{8}{3}}$, 3

evidence of approach *(M1)*

e.g. $f'(a) = \text{gradient}$, $3a^2 = \dfrac{-8}{-2-\frac{2}{3}}$, $\dfrac{a^3}{a-\frac{2}{3}} = 3$

simplify *A1*

e.g. $3a^2 = 3$, $a^2 = 1$

$a = 1$ *AG* *N0*

[7 marks]

(b) approach to find area of T involving subtraction and integrals *(M1)*

e.g. $\int f-(3x-2)\,\mathrm{d}x$, $\int_{-2}^{k}(3x-2) - \int_{-2}^{k}x^3$, $\int(x^3-3x+2)$

correct integration with correct signs *A1A1A1*

e.g. $\frac{1}{4}x^4 - \frac{3}{2}x^2 + 2x$, $\frac{3}{2}x^2 - 2x - \frac{1}{4}x^4$

correct limits -2 and k (seen anywhere) *A1*

e.g. $\int_{-2}^{k}(x^3-3x+2)\,\mathrm{d}x$, $\left[\frac{1}{4}x^4 - \frac{3}{2}x^2 + 2x\right]_{-2}^{k}$

attempt to substitute k and -2 *(M1)*

correct substitution into **their** integral if 2 or more terms *A1*

e.g. $\left(\frac{1}{4}k^4 - \frac{3}{2}k^2 + 2k\right) - (4-6-4)$

setting **their** integral expression equal to $2k+4$ (seen anywhere) *(M1)*

simplifying *A1*

e.g. $\frac{1}{4}k^4 - \frac{3}{2}k^2 + 2 = 0$

$k^4 - 6k^2 + 8 = 0$ *AG* *N0*

[9 marks]

Total [16 marks]